# Chem 532: Problem Set #4

Due in class: Monday, Oct. 17

**(1)** A particle in a spherically symmetric potential is known to be in an eigenstate of $\ell^2$ and $\ell_z$ with eigenvalues $\hbar^2\ell(\ell+1)$ and $m\hbar$, respectively. Show that the expectation values between $|\ell m\rangle$ eigenstates satisfy

$\langle \ell_x \rangle = \langle \ell_y \rangle = 0$ and

$$\langle \ell_x^2 \rangle = \langle \ell_y^2 \rangle = \frac{\left[\ell(\ell+1)\hbar^2 - m^2\hbar^2\right]}{2}$$

**(2)** Using the real-valued spherical harmonics defined as

$$Y_x = \frac{1}{\sqrt{2}}\left(Y_{1,-1} - Y_{1,1}\right)$$
$$Y_y = \frac{i}{\sqrt{2}}\left(Y_{1,1} + Y_{1,-1}\right)$$
$$Y_z = Y_{1,0}$$

evaluate each of the following:

**(a)** $\langle Y_y | \ell_y | Y_y \rangle$

**(b)** $\langle Y_y | \ell_z | Y_y \rangle$

**(c)** $\langle Y_z | \ell_x | Y_z \rangle$

**(d)** $\langle Y_z | \ell_x{}^2 + \ell_y{}^2 | Y_z \rangle$

**(3)** Locate the radial nodes of the 3*s* orbital of the hydrogen atom. How do these compare to those of $Ar^{17+}$?

**(4)** Calculate (a) the mean radius, (b) the mean square radius, and (c) the most probable radius of the 2*s* orbital of a hydrogenic atom of atomic number *Z*. You will find the following integral useful:

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$