HW #7

48 total

(10) ① Term symbols

a) [Ar] $4s^2 3d^9 4p$

$\left.\begin{array}{l} l_1 = 2 \\ l_2 = 1 \end{array}\right\} L = 3, 2, 1$ $\quad$ ${}^3F, {}^3D, {}^3P$

$\left.\begin{array}{l} s_1 = 1/2 \\ s_2 = 1/2 \end{array}\right\} S = 1, 0$ $\quad$ ${}^1F, {}^1D, {}^1P$

$J = L + S$: ${}^3F_4, {}^3F_3, {}^3F_2, {}^1F_3$

${}^3D_3, {}^3D_2, {}^3D_1, {}^3D_2$

${}^3P_2, {}^3P_1, {}^3P_0, {}^1P_1$

b) [Xe] $6s\,5d\,4f$

$\left.\begin{array}{l} l_1 = 0 \\ l_2 = 2 \end{array}\right\} L = 2$ couple w/ $l_3 = 3 \rightarrow L = 5, 4, 3, 2, 1$

$\left.\begin{array}{l} s_1 = 1/2 \\ s_2 = 1/2 \end{array}\right\} S = 1, 0$ couple w/ $s_3 = 1/2 \rightarrow \frac{3}{2}, \frac{1}{2}, \frac{1}{2}$

$J = L + S$ (lots of them)

${}^4H_{13/2}, {}^4H_{11/2}, {}^4H_{9/2}, {}^4H_{7/2}$

$({}^2H_{11/2}, {}^2H_{9/2}) \times 2$

${}^4G_{11/2}, {}^4G_{9/2}, {}^4G_{7/2}, {}^4G_{5/2}$

$({}^2G_{9/2}, {}^2G_{7/2}) \times 2$

${}^4F_{9/2}, {}^4F_{7/2}, {}^4F_{5/2}, {}^4F_{3/2}$

$({}^2F_{7/2}, {}^2F_{5/2}) \times 2$

${}^4D_{7/2}, {}^4D_{5/2}, {}^4D_{3/2}, {}^4D_{1/2}$

$({}^2D_{5/2}, {}^2D_{3/2}) \times 2$

${}^4P_{5/2}, {}^4P_{3/2}, {}^4P_{1/2}$

$({}^2P_{3/2}, {}^2P_{1/2}) \times 2$

(2) # of states

(8)

a) ^4F $J = \frac{9}{2}, \frac{7}{2}, \frac{5}{2}, \frac{3}{2}$

states $= 2J+1 = 10 + 8 + 6 + 4 = 28$

b) ^1S $J = 0$ $\rightarrow$ 1 state

c) ^3P $J = 2, 1, 0$

states $= 5 + 3 + 1 = 9$ states

d) ^2D $J = \frac{5}{2}, \frac{3}{2}$

states $= 6 + 4 = 10$ states

(3) ground states via Hund's rules

(10)

(a) $[Ar]4s^2 4p^3$

max S : $S = 3/2$

for this S, max $L = 0$ $\left(\sum m_{\ell i} = 0\right)$

$^4S_{3/2}$

(b) $[Kr]\, 5s^2 4d^2$

max $S = 1$ (triplet)

max $L = 3$ from $m_{\ell_1} = 2$ $m_{\ell_2} = 1 \Rightarrow M_L = 3$

^3F w/ $J = 4, 3, 2$

since less than 1/2 filled shell $\rightarrow$ $\boxed{^3F_2}$

(3) [Xe]4f5d6s

(4)

a) $\left.\begin{matrix} l_1 = 3 \\ \left.\begin{matrix} l_2 = 2 \\ l_3 = 0 \end{matrix}\right\rangle 2 \end{matrix}\right\rangle L = 5, 4, 3, 2, 1$

(5) $s_1 = s_2 = s_3 = 1/2$ $\quad \left.\begin{matrix} s_1 + s_2 = 1, 0 \\ s_3 = 1/2 \end{matrix}\right\rangle S = \frac{3}{2}, \frac{1}{2}, \frac{1}{2}$

Terms: ${}^4H, {}^2H, {}^2H, {}^4G, {}^2G, {}^2G, {}^4F, {}^2F, {}^2F, {}^4D, {}^2D, {}^2D,$
${}^4P, {}^2P, {}^2P$

Levels for highest L & S: ${}^4H_{13/2}, {}^4H_{11/2}, {}^4H_{9/2}, {}^4H_{7/2}$

b) for the ${}^4H_{13/2}$ level

$$L^2\psi = \hbar^2 L(L+1)\psi = \hbar^2 (5)(5+1)\psi$$

(5)

$$\langle L^2 \rangle = 30\hbar^2$$

$$\langle S^2 \rangle = \hbar^2 S(S+1) = \hbar^2 \left(\frac{3}{2}\right)\left(\frac{3}{2}+1\right) = \frac{15}{4}\hbar^2$$

$$\langle J^2 \rangle = \hbar^2 \left(\frac{13}{2}\right)\left(\frac{13}{2}+1\right) = \frac{195}{4}\hbar^2$$

(c) For $m_J = 13/2$, $m_S = 3/2$ and $M_L = \frac{10}{2} = 5$

therefore the possible spin-orbitals are

$$5d_2\,\alpha \quad \leftarrow m_l = 2$$

(5)

$$4f_3\,\alpha \quad \leftarrow m_l = 3$$

$$6s\,\alpha \quad \leftarrow m_l = 0$$

$$\uparrow \qquad\qquad \overline{}$$

$$M_S = 3\left(\tfrac{1}{2}\right) = 3/2 \qquad M_L = 5$$

$$\psi = \frac{1}{\sqrt{3!}} \begin{vmatrix} 4f_3(1)\alpha(1) & 5d_2(1)\alpha(1) & 6s(1)\alpha(1) \\ 4f_3(2)\alpha(2) & 5d_2(2)\alpha(2) & 6s(2)\alpha(2) \\ 4f_3(3)\alpha(3) & 5d_2(3)\alpha(3) & 6s(3)\alpha(3) \end{vmatrix}$$

(d) $S_-\left|s = 3/2, m_s = 3/2\right\rangle = \hbar\sqrt{\tfrac{3}{2}\left(\tfrac{3}{2}+1\right) - \tfrac{3}{2}\left(\tfrac{3}{2}-1\right)}\left|S = 3/2, m_s = 1/2\right\rangle$

$$= \hbar\sqrt{3}\left|3/2, 1/2\right\rangle$$

(5)

$$\left(s_{1-} + s_{2-} + s_{3-}\right)\left|\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right\rangle = \hbar\left|-\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \hbar\left|\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \hbar\left|\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

so $\left|\tfrac{3}{2}, \tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left[\left|-\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \left|\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \left|\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}\right\rangle\right]$

$$= \frac{1}{\sqrt{3}}\left[\beta(1)\alpha(2)\alpha(3) + \alpha(1)\beta(2)\alpha(3) + \alpha(1)\alpha(2)\beta(3)\right]$$