Chem 534: Problem Set #6

Due in class: Tues, Nov. 3rd

From McQuarrie, Statistical Mechanics:

A modification of the Debye theory was introduced by Born, who proposed a different cutoff for the spectrum of vibrational modes. He proposed that the cutoff be made such that both the longitudinal and transverse modes have a common minimum wavelength. If we denote this common minimum wavelength by λ_m, then $\lambda_m \nu_{long} = c_{long}$ and $\lambda_m \nu_{trans} = c_{trans}$, the normalization of $g(\nu)$ becomes:

$$4\pi V \left\{ \int_0^{\nu_t} \frac{2}{c_{trans}^3} \nu^2 \, d\nu + \int_0^{\nu_l} \frac{1}{c_{long}^3} \nu^2 \, d\nu \right\} = 3N$$

Show that this leads to the following expression for the heat capacity:

$$C_v = R\left[D\left(\frac{\Theta_l}{T}\right) + 2D\left(\frac{\Theta_t}{T}\right) \right]$$ where $D(z)$ is the Debye function: $D(z) = \frac{3}{z^3}\int_0^x \frac{x^4 e^x dx}{\left(e^x - 1\right)^2}$, where

$$x = \frac{h\nu}{kT}$$