Chem 332: Problem Set #2

<u>Due in class: Friday, Jan. 25th</u>

(1) In normalizing wavefunctions, the integration is over all space in which the wave function is defined. Normalize the following wavefunctions.

(a) $\psi = e^{-x^2/2}$ over the range $-\infty \le x \le +\infty$

(b) $\psi = \sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{b}y\right)$ over the range $0 \le x \le a$ and $0 \le y \le b$, where a and b are constants and n and m are integers.

(2) A quantum mechanical particle confined to move in one dimension between $x = 0$ and $x = L$ is found to have a state described by the wavefunction

$$\psi(x) = A\sin\left(\frac{2\pi}{L}x\right).$$

(a) Determine the constant A such that the wavefunction is normalized.

(b) Using the result of part (a), find the probability that the particle will be found between $x = 0$ and $x = L/3$.

(3) Determine in each of the following cases if the function in the first column is an eigenfunction of the operator in the 2nd column. If so, state what the eigenvalue is.

Function	Operator
$\cos\theta$	$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right)$
$e^{-i\left(2ix^2\right)}$	$\frac{d^2}{dx^2}+16x^2$
$\cos x \sin x$	$\frac{d^2}{dx^2}-2$

(4) Using Euler's relation, $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$,

(a) Verify $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$ and $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$

(b) Using the expression for $\cos\theta$ in part (a), show that $\frac{d(\cos\theta)}{d\theta} = -\sin\theta$.

(5) In classical mechanics the angular momentum vector $\vec{L}$ is defined by $\vec{L} = \vec{r} \times \vec{p}$. Determine the x component of $\vec{L}$ in terms of the components of $\vec{r}$ and $\vec{p}$, i.e., (x, y, z) and (p_x, p_y, p_z). Now apply the Schrödinger prescription to show that the quantum mechanical operator for L_x is $\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$.

(6) Show that the linear combination $\hat{B} + i\hat{C}$ is <u>not</u> hermitian if $\hat{B}$ and $\hat{C}$ <u>are</u> hermitian operators.