Chem 332: Problem Set #3

Due in class: Friday, Feb. 1st

(1) The standard deviation of a measurement can be translated into quantum mechanics as $\sigma_A = \sqrt{\left\langle \left[\hat{A} - \langle \hat{A} \rangle\right]^2 \right\rangle}$, or equivalently $\sigma_A = \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2}$, i.e., the square root of the difference in the expectation value of $\hat{A}^2$ and the square of the expectation value of just $\hat{A}$, where $\hat{A}$ is the hermitian operator associated with the observable A. Consider a particle in a state described by the wavefunction $\psi_0 = \left(\frac{a}{\pi}\right)^{1/4} e^{-x^2/2}$, where a is a constant and $-\infty \le x \le \infty$. Calculate the standard deviation of the particle's momentum .

(2) An excited state wavefunction (a higher energy solution of the Schrödinger equation) associated with the same system as problem (1) is found to be

$$\psi_2 = \left(\frac{a}{\pi}\right)^{1/4} \frac{1}{\sqrt{2}}\left(2x^2 - 1\right)e^{-x^2/2}$$

Show that ψ_0 and ψ_2 are orthogonal to each other.

(3) A particle is in a state described by the wavefunction $\psi = \frac{1}{\sqrt{3}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2 + \frac{1}{\sqrt{6}}\phi_3$, where ϕ_n are normalized solutions to the Schrödinger equation for this particular system, i.e., $H\phi_n = E_n\phi_n$ with $\phi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)$ and corresponding eigenvalues $E_n = \frac{n^2h^2}{8mL^2}$.

(a) What are the possible outcomes for a single measurement of the total energy E and what are the probabilities of obtaining each one?

(b) What is the expectation value of the Hamiltonian for this system? (remember that $\langle \hat{H} \rangle$ is the average total energy)

(4) Determine the commutators of the following pairs of operators.

(a) $\frac{d}{dx}$ and $\frac{1}{x}$

(b) a and $a^{\dagger}$, where $a = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{p})$ and $a^{\dagger} = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{p})$ with $\hat{p} = -i\hbar \frac{d}{dx}$.