

## The Temperature Dependence of $\ln K$

van't Hoff equation: 
$$\frac{d \ln K}{dT} = \frac{\Delta H_r^\circ}{RT^2}$$

**What if we can't assume that  $\Delta H_r^\circ$  is independent of T?**

Recall: 
$$\Delta H_r^\circ(T) = \Delta H_r^\circ(T=0) + \int_0^T \Delta_r C_p \quad (\text{assuming no phase changes})$$

where 
$$\Delta_r C_p = \sum_i^{N_s} \nu_i C_{p,i}$$

and in general,  $C_p = a + bT + cT^2 + \dots$  and then  $\Delta_r C_p = \Delta a + \Delta bT + \Delta cT^2 + \dots$

(and  $\Delta b = \sum_i^{N_s} \nu_i b_i$ , etc.)

Then:

A. 
$$\Delta H_r^\circ(T) = \Delta H_r^\circ(T=0) + \Delta aT + \frac{1}{2}\Delta bT^2 + \frac{1}{3}\Delta cT^3 + \dots$$

and: 
$$d \ln K = \frac{1}{RT^2} \left[ \Delta H_r^\circ(T=0) + \Delta aT + \frac{1}{2}\Delta bT^2 + \frac{1}{3}\Delta cT^3 + \dots \right] dT$$

integrating:

B. 
$$\ln K = \Delta \frac{\Delta H_r^\circ(T=0)}{RT} + \frac{\Delta a}{R} \ln T + \frac{\Delta b}{2R} T + \dots + I$$

where  $I$  is a constant of integration

Notes:

- (1) if  $\Delta H_r^\circ(T)$  is known for any one temperature (perhaps 298 K) and  $C_p(T)$  is known for each reactant and product, then  $\Delta H_r^\circ(T=0)$  can be calculated (eqn. A).
- (2) if  $\Delta H_r^\circ(T=0)$  is known for the reaction and  $K$  is known for any one temperature,  $I$  can be evaluated (eqn. B).
- (3) Then  $K$  can be calculated at any temperature.