

THE POSTULATES OF QUANTUM MECHANICS

Postulate I (The system is described by a wave function)

Any bound state of a dynamical system of n particles is described as completely as possible by an acceptable, square-integrable function $\Psi(q_1, q_2, \dots, q_{3n}; \omega_1, \omega_2, \dots, \omega_n; t)$, where the q 's are spatial coordinates, ω 's are spin coordinates, and t is the time coordinate. $\Psi^*\Psi d\tau$ is the probability that the space-spin coordinates lie in the volume element $d\tau (\equiv d\tau_1 d\tau_2 \dots d\tau_n)$ at time t , if Ψ is normalized.

Postulate II (Physical observables are associated with hermitian operators)

To every observable dynamical variable M (classical physical observable), we associate a hermitian operator \hat{M} by :

- (1) write the classical expression as fully as possible in terms of cartesian momenta and positions
- (2) if M is q or t , \hat{M} is q or t
- (3) if M is a momentum, p_q , the operator is $-i\hbar \frac{\partial}{\partial q}$, where q is conjugate to p (e.g., x is conjugate to p_x).
- (4) If M is expressible in terms of q 's, p 's, and t , \hat{M} is found by substituting the above operators in the expression for M . Nearly always this will provide a hermitian operator.

Postulate III (Wave functions are solutions of the TDSE)

The state functions (or wavefunctions) satisfy the time dependent Schrödinger equation

$$\hat{H}\Psi(q, t) = i\hbar \frac{\partial}{\partial t} \Psi(q, t)$$

where \hat{H} is the hamiltonian operator for the system.

Postulate IV (Precise measurements: eigenvalues/eigenfunctions)

If Ψ_b is an eigenfunction of the operator \hat{B} with eigenvalue b , then if we make a measurement of the physical observable represented by \hat{B} for a system whose wavefunction is Ψ_b , we always obtain b as the result.

Postulate V (Imprecise measurements: average or expectation values)

When a large number of identical systems have the same wavefunction Ψ , the expected average (“expectation value”) of measurements on the observable M (one measurement per system) is given by

$$\langle M \rangle = \frac{\int \Psi^* \hat{M} \Psi d\tau}{\int \Psi^* \Psi d\tau}$$

(The denominator equals one if Ψ is normalized.)

Note that if Ψ is an eigenfunction of the operator \hat{M} , this postulate reverts to Postulate IV.